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Multiple time scales in serial production of force: A tutorial on power spectral analysis of motor variability

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Abstract

We present a tutorial on a power spectral approach to variability in serial motor performance, describing as a case study two experiments on the form of the variance in two force production tasks. In Experiment 1 we examine grip force and load force in repetitive unimanual pulling; in Experiment 2, we describe repetitive bimanual pressing. In both experiments log–log plots of power spectral density of peak force of the responses in each stream against frequency (i.e. periodicity or repetition cycle defined with respect to the ordered succession of responses) were approximately linear with negative slopes which varied systematically with test conditions. We propose the two response streams in each experiment are associated with different levels of sustained attention and state a simple model involving summation of moving average processes on multiple time scales as a qualitative account of the changes in $1/f$ slope.

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1. Introduction

Repeated movements are rarely, if ever, exactly the same but are subject to variation. In some cases, the variation may be intended, as in the same musical phrase repeated at different points in a performance with different timing and dynamic (loudness) for expressive contrast (Shaffer, 1981). However, the variation may be involuntary and a reflection of the limits of skill. For instance, in repetitive tapping the intervals between tap responses exhibit variance that increases with the mean (Wing & Kristofferson, 1973a). Such dependence of performance variability on the mean, comprises signal dependent noise, and may contribute to the strategic nature of motor control in hand positioning (Meyer, Smith, Kornblum, Abrams, & Wright, 1990; Meyer, Smith, & Wright, 1982) and eye movements (Harris & Wolpert, 1998). In the case of motor timing, signal dependent noise may contribute to a strategy in synchronising tapping with a metronome, in which the response leads the pacing stimulus (Vorberg & Wing, 1996).

Just as the variability of movement can influence the nature of motor control, so can motor variability be an important indicator of the nature of control deserving explicit study (Wing, 1992). In this paper we provide a tutorial review with two experiments as a case study of processes with long-time scales that may underlie motor variability. The approach we take is based on spectral analysis of fluctuations in extended series of observations of a task involving force production. We first set the scene by considering the case of motor timing, a topic in which autocorrelation has played a significant role.

2. Autocorrelation function

Variability in movement may arise from many different sources. In studies of movement timing, a contrast has been drawn between variance arising in peripheral motor implementation and variance due to central timekeeping (Wing & Kristofferson, 1973a). An explicit model of the hierarchical relation between timekeeper intervals and motor implementation delays (Wing & Kristofferson, 1973b) predicts a particular form of variability in repetitive responding. In repetitive tapping, successive interresponse intervals should be negatively correlated, whereas intervals separated by more than one intervening interval should be uncorrelated. Thus, in considering the variability of movement it is important to observe the nature of variance and, in the case of serial timing behaviour, to characterise sequential dependence in series of interresponse intervals.

The autocorrelation function is often used to describe sequential dependence in time series data. For example the Wing–Kristofferson (WK) model predicts a negative autocorrelation bounded by zero and minus one half at lag one, and zero autocorrelations at higher lags. The WK model is an example of a form of random process, termed first-order moving average, MA(1). The model assumes that the current interresponse interval includes the sum of two random terms, reflecting delays in implementing the responses initiating and terminating the interval (plus another random term reflecting timekeeper variance). The short-term dependence of the first-order moving average process may be contrasted with long-term dependence produced by another generating process, the first-order autoregressive process, AR(1) (for an introduction to autoregressive and moving average processes see Chatfield, 2003). In an AR(1) process the current value involves the sum of two random terms, one of which is a proportion of the random term associated with the previous value. Such a process results in an autocorrelation function with nonzero values persisting to long lags. An example of timing behaviour which results in an autoregressive process is synchronisation of responding with a metronome. Adjustment of the phase between responses and metronome pulses has been modelled as a first-order autoregressive process with proportional error correction (Pressing, 1998a; Vorberg & Schulze, 2002; Vorberg & Wing, 1996). However, in this approach, implementing the phase correction invokes additional random terms yielding a mixed first-order autoregressive – moving average model (ARMA(1,1) in the terminology of Box & Jenkins, 1976).

We have already observed that, in sequential timing tasks, the variability increases with the mean. However, it has also been noted that there may be differences in the form of variability at shorter and longer intervals. For instance, Stevens (1886) noted longer-term fluctuations when longer intervals had to be produced. Madison (2001) examined a tendency of the interresponse interval to drift in the mean and reported that this is more pronounced when the number of intervals to be produced increases. He proposed a method of differencing successive interresponse intervals to reveal the time scale of the drift. Recently, Ogden and Collier (1999, 2002; see also Collier & Ogden, 2004) have proposed a differencing method as a means of estimating a component of variance attributable to drift separate from estimates of clock and implementation delay variance in the WK model.

In summary, the pattern of variability in simple discrete timing may be understood in terms of two or more processes generating clock and implementation delays in a hierarchical control structure (Pressing, 1998a; Vorberg & Wing, 1996; Wing, 1980; Wing & Kristofferson, 1973b). Implicit in the concept of two levels of control in timing is the idea that the component processes have different time scales. Whereas motor implementation comprises an automatic process running on a short-time scale (the current and previous response in the WK model), central timing might be expected to involve cognitive processes such as attention and working memory (Sergent, Hellige, & Cherry, 1993; Treisman, 1963) that run on a longer-time scale than motor execution.

So far we have discussed forms of variability in time series in terms of the autocorrelation function. The autocorrelation function is equal to the autocovariance function normalised by the variance. Recently, the power spectrum has been proposed as a means of characterising fluctuations in timing behaviour (Gilden, Thornton, & Mallon, 1995). The power spectrum is the Fourier transform of the autocovariance function. Using the power spectrum constitutes a frequency-based approach that is complementary to the time-based approach based on the autocovariance function (Box & Jenkins, 1976; for the relation between autocovariance and power spectrum in relation to timing, see Pressing & Jolley-Rogers, 1997).

The power spectral density function has useful properties in the context of long-range dependence in serial performance that we now consider.

3. Power spectral density

The power spectrum of a series of N discrete behavioural measurements $x_1, x_2, x_3, \ldots, x_N$ is the Fourier-transform of its autocorrelation function. An autocorrelation function that decays like $\tau^{-\gamma}$ corresponds to a spectral density of the form $S(f) \propto f^{\gamma-1} = f^{-\beta}$. The (discrete) Fourier transform of a sequence $\{x_t\}$ is defined as

$$X(f) = \sum_{t=1}^{N} x_t \mathrm{e}^{-\mathrm{i}2\pi ft}$$

and $X(f)X^*(f) = \|X(f)\|^2$ represents the power spectrum, which can be extended to the power spectral density $\|X(f)\|^2 \rightarrow S(f)$ by using, for instance Welch's periodogram method (Hayes, 1996). The quantity $S(f)df$ represents the contribution to the spectral power of the set $\{x_t\}$ made by frequencies in the range $(f, f + df)$. For $S(f) \propto f^{\gamma-1} = f^{-\beta}$ in which γ is very small, that is, in cases where the autocorrelation decays slowly indicating long-term correlations, the exponent β in the power spectral density will be about 1 and the underlying (random) process is referred to as "1/*f* noise" (also "flicker noise" or "pink noise"). Put differently, 1/*f* noise denotes a relation between a system's power spectral density $S(f)$ and its frequency f in the form of a power law with a characteristic exponent β which, in practice, may vary as widely as -0.5 to -1.5. [1] Hence, a log–log plot of S as a function of f will be linear with slope $-\beta$.

1/*f* type noise occurs widely in physical systems such as semiconductors, superconductors and optical devices (for a recent review see Sornette, 2004). Evidence of 1/*f* noise has also been found in human time estimation, (Ding, Chen, & Kelso, 2002; Farrell, Wagenmakers, & Ratcliff, submitted; Gilden et al., 1995; Madison, 2004; Pressing & Jolley-Rogers, 1997) and in sequential reaction time (RT) measurements of a variety of cognitive processes including mental rotation, lexical decision, serial visual search, and parallel visual search (Gilden, 1997) and word naming (Van Orden, Holden, & Turvey, 2003).

A variety of different mechanisms can give rise to 1/*f* spectral plots (Handel & Chung, 1993; Kawai, Mihira, Sato, & Hayashi, 1993; Miller, Miller, & McWhorten, 1993; Montroll & Shlesinger, 1982; Rangarajan & Ding, 2003; Sornette, 2004; Weissman, 1988). However, two major themes recur. Theme one suggests that 1/*f* noise arises due to certain kinds of system instability. In particular, deterministic nonlinear systems operating in specific regimes of chaos or intermittency often yield spectra of this form (Bak, Tang, & Weisenfeld, 1987; Handel & Chung, 1993; Schuster, 1996). Linear systems operating on the cusp of instability can also produce straight-line

[1] Along with β or γ, the Hurst exponent H frequently serves to quantify power laws. It relates to the values used here by means of $H = -\frac{1}{2}\gamma = \frac{1}{2}(\beta + 1)$.

log–log plots, but slopes are generally considerably more positive than -1, and may show a high-frequency rise (Pressing, 1998b). A second theme ascribes $1/f$ noise to self-similarity of control or to the phenomenon of scale invariance because, in the limiting case of a $1/f$ form that persists over (infinitely) many octaves of frequency, the system exhibits an absence of a characteristic distance or time scale. Human behavioural data, however, usually span a fairly limited range of frequencies (2–3 octaves) so that ascribing general scale invariance by means of infinitely many time scales is decidedly speculative.

4. Multiple time scales

As we will show in the following, a few specific time scales may be all that is needed to produce $1/f$ spectra. Indeed, $1/f$ spectra can arise from systems that operate over a finite number of time scales, e.g., as seen in process latencies or relaxation times. In accord with this, it has been shown that certain continuous distributions of multiple time scale processes readily yield $1/f$ spectra (Montroll & Shlesinger, 1982; Handel & Chung, 1993). Moreover Pressing (1998b) showed that simple linear sums of moving averages of random sources using only 2–3 distinct time scales can produce $1/f$ spectra (see also Alessio, Carbone, Castelli, & Frappietro, 2002).

In more detail, when considering an uncorrelated white noise process x_t (see Fig. 1), one may generate multiple time scales from such a process using a moving average operation. To illustrate this procedure we denote the moving average of x_t based on a time window of T points by $\langle x_t \rangle_T$, that is, the tth term of $\langle x_t \rangle_T$ is just

$$\langle x_t \rangle_T = \frac{x_t + x_{t-1} + x_{t-2} + \cdots + x_{t-T+1}}{T} = \frac{1}{T}\sum_{\Delta=0}^{T-1} x_{t-\Delta}$$

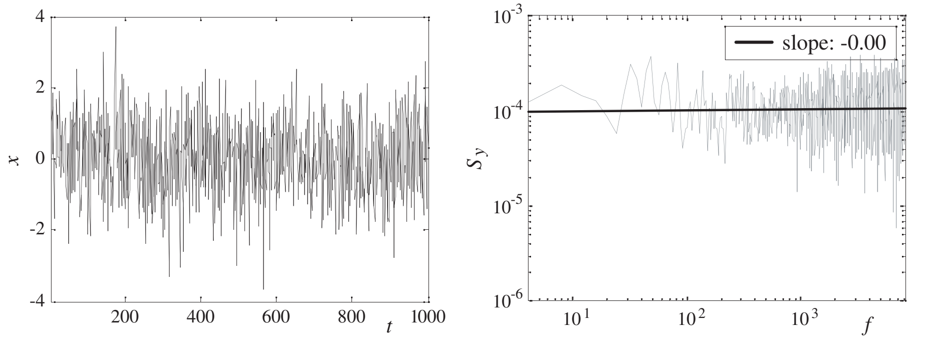

Fig. 1. Sequence generated by a random process (Gaussian white noise with vanishing mean and unit variance; left panel) and its power spectral density (right panel). The sequence contains 2^{14} samples and $S(f)$ has been computed using Welch's periodogram method with overlapping Hamming windows of $\frac{1}{4}$ of the sequence's length. Since the time series does not contain any correlations the slope of $S(f)$ is zero.

An example of a 3-time scale process would then be

$$X_t = x_t + w_1\langle x_t\rangle_{T_1} + w_2\langle x_t\rangle_{T_2}$$

which invokes the time scales 1, T_1 and T_2. Here w_1 and w_2 are weights for the moving average processes. Figs. 1–3 contrast the white noise process with two multiple time scale processes in terms of their time series and their resultant spectral power plots. It will be noted that the log–log plot of the power spectral density has slope zero in the case of the uncorrelated white noise process but has negative slope which changes with the number of time scales in the case of processes with multiple time scales.

A more systematic examination of the effects of parameters in multiple time scale processes was given in Pressing (1998b). There it was shown that spectral shape is not markedly affected by parameter values, provided that slow processes have greater

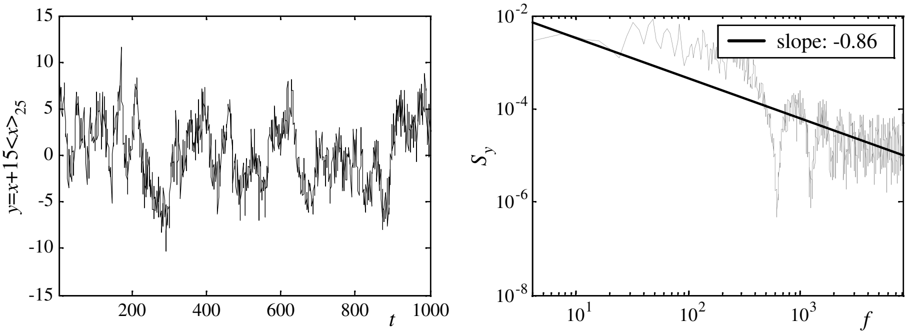

Fig. 2. Sequence of a random process (left panel) based on the Gaussian white noise shown in Fig. 1. A single moving average in the form of $y = x + 15\langle x\rangle_{25}$ causes a negative slope in the log–log plot of the power spectral density (right panel).

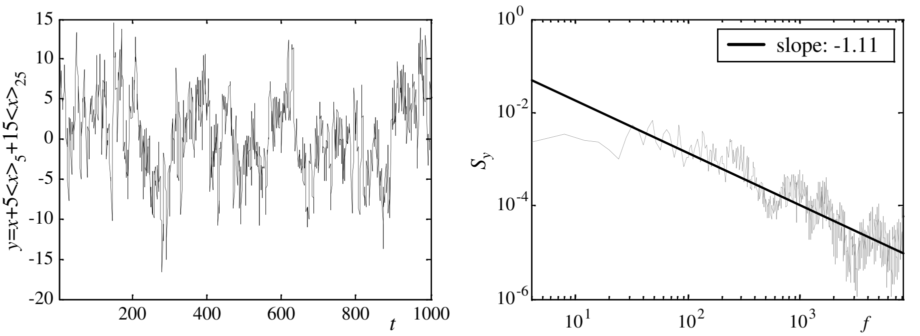

Fig. 3. Sequence of a three time scale random process (left panel; cf. Figs. 1 and 2): $y = x + 5\langle x\rangle_{15} + 15\langle x\rangle_{25}$. In comparison with a two time scale process (Fig. 2) the slope of the power spectral density is steeper, i.e. more negative (right panel).

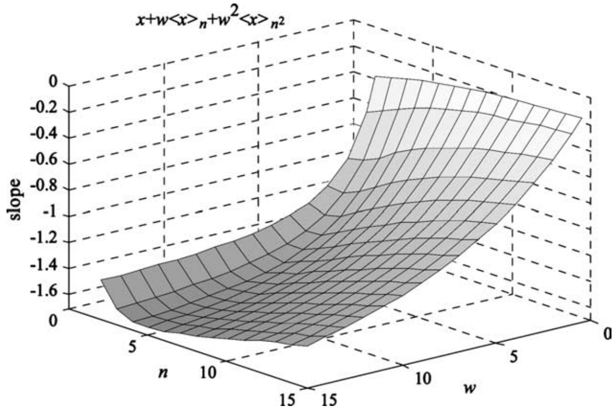

Fig. 4. Estimates of the linear slopes of three time scale random processes of the form $y = x + w\langle x\rangle_n + w^2\langle x\rangle_{n^2}$ for different parameters w and n. The slope is at a minimum for suitable combination of the two moving averages (here about $n \approx 5$) and decreases (becomes more negative) with increasing weighting factor w; see Pressing (1998b) for more details.

weights than fast processes (this basically means that fluctuations are typically larger at longer-time scales than they are at shorter time scales, a sensible condition for natural control systems). Exponents lie overwhelmingly in the range -1.5 to -0.5 when time scale factors are in the range 2–15, and weight factors are in the range 2–100. The value of the slope tends to increase (become less negative) with decreasing weights, or with increasing time scale factors (see Fig. 4).

Further work has shown that the use of uncorrelated random source processes (i.e. $x_t \sim$ white noise) is not essential to generate $1/f$ characteristics using moving averages. Broadband noise only is required, and the underlying system may be high dimensional stochastic or low dimensional deterministic (i.e. chaotic, since for the present purpose it is virtually identical to a white noise source). For example, moderate nonstationarity of parameters such as monotonic drift in the mean results in power spectral density with somewhat similar characteristics. However trend removal results in zero slope power spectral density whereas the slope of the long-time scale process is unaffected by trend removal (see Fig. 5).

5. Application of the power spectrum to task performance

In relation to the discussion above, we note that, if performance of cognitive tasks invokes processes such as memory and attention on multiple time scales, the tasks will show order, and correspondingly, the suppression of noise fluctuations, on multiple time scales; hence we might expect the noise to be of the $f^{-\beta}$ form. An indication that sustained attention operates on at least two time scales comes from an EEG study of fluctuations in performance on a vigilance task in which participants detected noise burst targets in a white noise background (Makeig & Inlow, 1993).

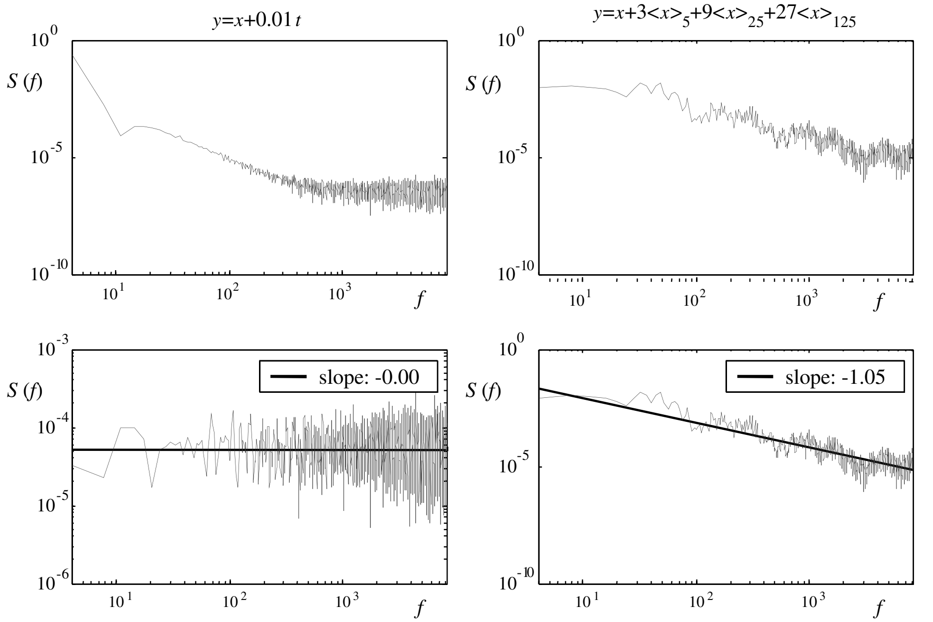

Fig. 5. Comparison between long-term correlation processes (right column) and linear drift effects (left column) with estimates of the power spectral density using the raw data (upper row) versus the piecewise linearly detrended data (lower row). Although at a first glance the power spectral density of a process with linear trend (left) appears to reflect a power law in the log–log representation of the spectral power albeit with a high frequency plateau, removing the trend flattens the slope and the lack of correlations is revealed. In contrast, power spectral density of the three time scale moving average process (right) is not affected by trend removal.

The mean rate of the targets was 10/min and it was found that local error rate defined over a 33 s moving window fluctuated widely. Coherence was observed between local error rate and EEG power in several frequency bands and this occurred over two cycle lengths with period 1.5 and 4 min, time scales considerably longer than the average response interval of 6 s. Brain damage affecting frontal regions is commonly characterised as resulting in problems of sustained attention (Stuss, Shallice, Alexander, & Picton, 1995). Subsequently, Stuss, Murphy, Binns, and Alexander (2003) showed that this results in greater variability of RT in choice decision tasks in patients with dorsolateral frontal lesions compared to patients with lesions in other areas. Lag one autocorrelation and trend tests did not reveal any systematic differences between patient groups but it is interesting to ask whether there might have been differences in 1/*f* noise with differences in sustained attention resulting in changes in multiple time scale processes.

Gilden (2001) reported 1/*f* noise in RT data from normal participants performing long series of trials in lexical decision and visual search tasks. Given the same form of variability in RT across different tasks, Gilden argued 1/*f* noise reflects processes involved in maintaining mental set over a set of trials, an operation that might reasonably be identified with sustained attention. He reported power spectra with zero slopes from an experiment with interleaved tasks as evidence that breaking set disrupts the long-time scale memory processes whose correlations create $1/f$ spectra. He also showed the presence of $1/f$ noise in pure motor tasks, including reproduction of distance, angle or force, and argued that such long-time scale processes are important even in the maintenance of a single intention. Subsequently, Van Orden et al. (2003) reported that simple vocal RT and choice (word naming) RT tasks both gave rise to similar $1/f$ noise functions. Although they took issue with Gilden's theoretical position, the essence of their account, that $1/f$ noise reflects near criticality in intentional states which might lead to one or other course of action, does bear a strong resemblance to Gilden's proposal. We propose that both views might be subsumed under the umbrella of processes supporting sustained attention.

6. Force production

In this tutorial we consider the case of force control in two studies designed to investigate the contribution of sustained attention to motor performance in extended series of responses. In each study the task required participants exert control over not just one performance dimension, as in the previously cited work, but on two dimensions, of which one is reasonably considered to be the more primary in terms of focus of attention. In Study I a unimanual force production task was used to compare simultaneous automatic and conscious aspects of motor control. The task involved using precision grip to repeatedly produce a target pull force on a manipulandum. Stability of grip under tangential load force associated with lifting an object involves anticipatory rise in grip force normal to the grasp surfaces (Johansson & Westling, 1984). We expected that in our task the demand of producing a pull force to a target level would be the primary focus of conscious attention whereas anticipatory rise in grip force would be more automatic and less under direct attentional control. Study II comprised a bimanual force production task. In Study II knowledge of results (KR) was provided for one hand. We expected that KR would focus conscious attention on that hand and there would be less attention to the other hand.

Consider the effects of sustained attention on the spectral plot. If attention is minimal, and operating purely locally, that is, over short-time scales only, then spectral fluctuations over the medium to long-term (medium to low frequencies) will not be suppressed. Consequently, there should be a rise in power at low frequencies, and an $f^{-\beta}$ straight line fit on a log–log plot will have a slope considerably less than zero. If, in comparison, task control operates to successfully limit fluctuations at a range of frequencies, via sustained attention (vigilance) across multiple time scales, spectral power of the process will be more uniformly distributed, and an $f^{-\beta}$ straight line fit on a log–log plot will produce a larger (less negative) slope. This is in accord with model simulations noted above (Pressing, 1998b) that show that the value of the slope tends to increase (become less negative) with decreasing weights (cf. Fig. 4). In other words, the fit flattens towards white noise as the contributions of different time scales become comparable. If we consider that the degree of sustained attention determines the relative weighting of time scales operating in process control, this leads to the following hypothesis. The slope of a 1/*f*-type log–log spectral plot (*S*) is an index of sustained attention; larger (less negative) slopes are an index of greater attention. In other words, *S* is a monotonic function of attention, and since slopes are generally negative, the steeper (more negative) the slope, the less effective is attention, and the smaller the number of time scales.

7. Study I: Unimanual pull task

7.1. Participants

Nine volunteers including the third author participated after providing informed consent. Seven were right handed, the others left-handed, for writing. The average age was 33 years (range 21–48 years).

7.2. Methods

Subjects used precision grip with opposed middle finger and thumb to perform repeated downward pulling actions on a manipulandum fixed to the underside of a table (see Fig. 6). The manipulandum comprised two load cells (F240, Novatech, Hastings, UK) mounted at right angles so that the lower one measured grip force between parallel, vertically oriented grip plates with smooth wood surfaces and the upper one measured load force between the upper transducer and the table. The grip span was 65 mm and the grip plates were rectangular $50 \times 25\,\text{mm}^2$. The load cells were interfaced to a Macintosh IIfx computer using an National Instruments A-to-D card (NBMIO16X) and force measures were sampled at 120 Hz.

Subjects were instructed to grip the plates with sufficient force to avoid slipping. Practice, with real-time display of the pull force, was given to establish target lower and higher pull force levels, using either the preferred or non-preferred hand. During

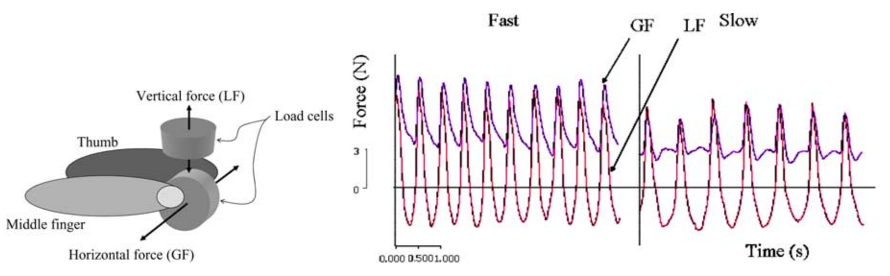

Fig. 6. Schematic of grip force apparatus showing manipulandum held in precision grip while producing vertical downward force pulses. The upper load cell registers vertical load while the lower load cell measures grip force, normal to the manipulandum surface. This generates friction and prevents slipping during each downward force pulse. Illustrative data from 5 s portions of 2 trials produced in fast (left) and slow (right) heavy conditions are shown on the right.

data collection trials there was no visual display and subjects were instructed not to look at the pulling hand to eliminate visual feedback.

Each trial began with a series of five computer-generated tones to establish the approximate speed. These tones stopped once the subject began to produce forces. A single trial comprised 550 force peaks. There were eight conditions, based on the three independent variables force target level, speed, and hand: low/high force (low = 3–4 N, high = 7–8 N), fast/slow speed (interval between force pulses approximately 300 ms or 600 ms), left or right hand. Order of conditions was counterbalanced across subjects. After the first four trials, subjects were allowed to observe a display of their force levels for the low and high force production values. If there was a substantial discrepancy from the target range, the subject was asked to correct the forces produced to be consistent with the starting range.

7.3. Analysis and results

Data collected were the digitally sampled values of force for the various conditions. Illustrative data from 5 s portions of two single trials are shown in Fig. 6.

Load and grip force peaks in the low-pass filtered (bi-directional Butterworth with 20 Hz cut-off) force records were determined by a peak-finding algorithm which first identified successive load force maxima and minima and then identified the maximum in grip force between each successive pair of load force minima. This yielded two time series of force peaks each comprising some 550 points. These data series were subjected to spectral analysis using periodogram estimators in MATLAB, and a conventional least squares straight line best fit on a log–log plot was used to determine the corresponding $1/f$ slope and intercept (the extrapolated power at zero frequency which, in combination with the slope, is a measure of overall variability assuming a straight line log–log periodogram) for the force and interval series (Fig. 7).

Repeated measures ANOVA revealed a significant difference ($F\ (1,8) = 14.925$, $p < 0.01$) in slope between load force and grip force, with the grip force slope being steeper (−0.607 versus −0.798) but no reliable main effects of hand or force level. There was a reliable interaction between force type and force level with no difference in load force slope but more negative grip force slope for the lighter pull than the heavier pull (−0.824 versus −0.753). Furthermore, there was a significant difference ($F\,(1,8) = 13.622$, $p < 0.01$) in intercept between load force and grip force with a larger (more positive) intercept for the latter (−2.851 versus −2.338). Finally, we found a significant interaction between force type and force level with a larger intercept for the heavier pull than for the lighter pull for grip force (−2.466 versus −2.210) but no differences in load force.

8. Study II: Bilateral press task

In the second study we sought to differentially manipulate control processes by providing KR on only one of two synchronous force pulse series. We were interested to determine whether spectral analysis on the time series of peak force values would

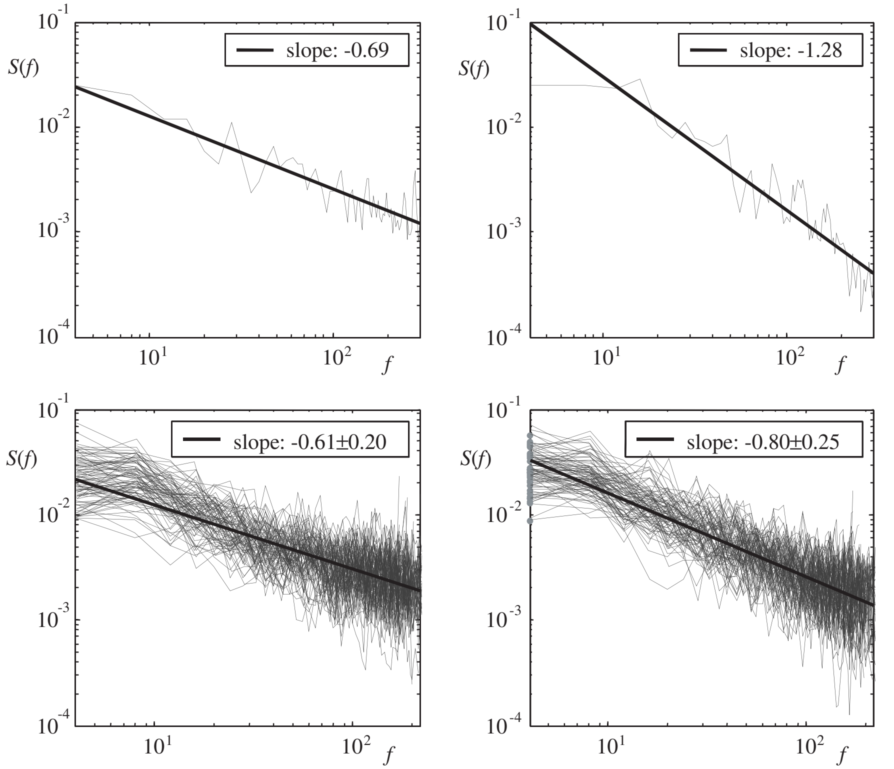

Fig. 7. (Above) Illustrative power spectral density functions for load force (left) and grip force (right) from one subject (single trial). Frequency is defined as the reciprocal of period with lower and upper limits set by half the sequence length and adjacent responses respectively. The best fit least squares regression line is superimposed on each function. (Below) Linear functions estimated for all subjects in Experiment 1 for load force (left) and grip force (right).

show decreasing power with frequency which was linear on a log–log plot. We expected that the slope would be greater (i.e. less negative) in the hand where KR was provided due to a memory time scale effect reflecting enhanced control processes operating over longer-time scales.

8.1. Subjects

Twelve volunteers including the first and third authors with average age 33 years (range 21–52 years) were tested. Four of the subjects were female, all were right-handed.

8.2. Methods

Subjects pressed down repeatedly with the index finger of each hand on a pair of load cells (F240, Novatech, Hastings, UK) interfaced with a MAC IIfx personal computer (as above) which sampled vertical force at 200 Hz. Each load cell was secured in front of the subject on either side of the midline so that, with forearm pronated, the thumb and middle through little fingers rested on the table on either side of the transducer while the index finger rested on a wood surface force plate on top of the transducer.

Subjects were instructed to produce pairs of simultaneous force pulses with the index fingers in response to a computer-generated tone occurring every 3–4 s. A trial consisted of about 250 tones in sequence. The instructions emphasised that attention was to be directed toward the right hand, which was to produce force pulses with peak force of 5 N. The left hand was also to produce force pulses in synchrony with the right, but without any attempt to monitor the level of force production. Two trials were run; on the first trial knowledge of results was provided; the force peak value of the right hand in each response appeared on a computer display immediately after the response, and remained visible until the next response. Subjects were instructed to monitor this display in order to improve task performance. No information was provided about left hand force. On the second trial, no information on the produced force levels was given for either hand.

8.3. Analysis and results

Peak force was determined for each pair of force pulses. Data from an illustrative trial are shown in Fig. 8.

Log–log spectral slopes for the two hands were computed as described above. Illustrative spectral density functions for the forces from one subject in the initial trial (with KR) are shown in Fig. 9 (above) separately for left hand (left) and right hand (right). In both plots a progressive decrease in power with frequency is shown

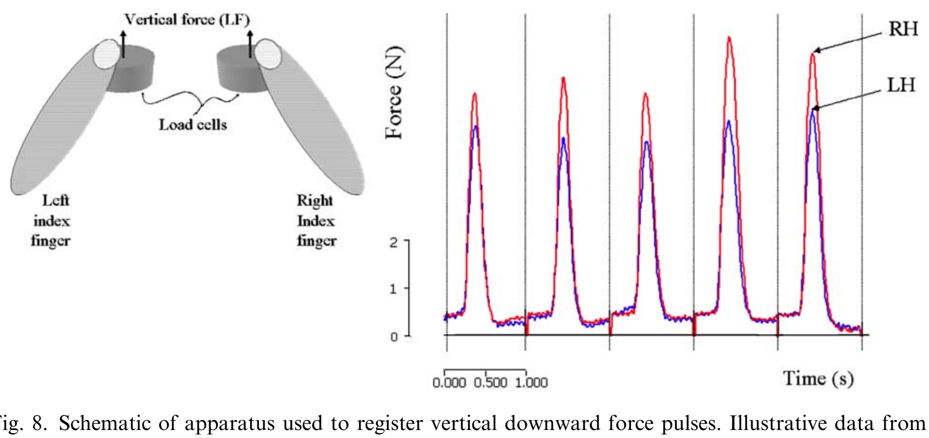

Fig. 8. Schematic of apparatus used to register vertical downward force pulses. Illustrative data from a series of 5 pulse pairs produced at the end of the second trial by one subjects are shown on the right. Each response pair was separated from the next by a brief pause indicated by the zero force values.

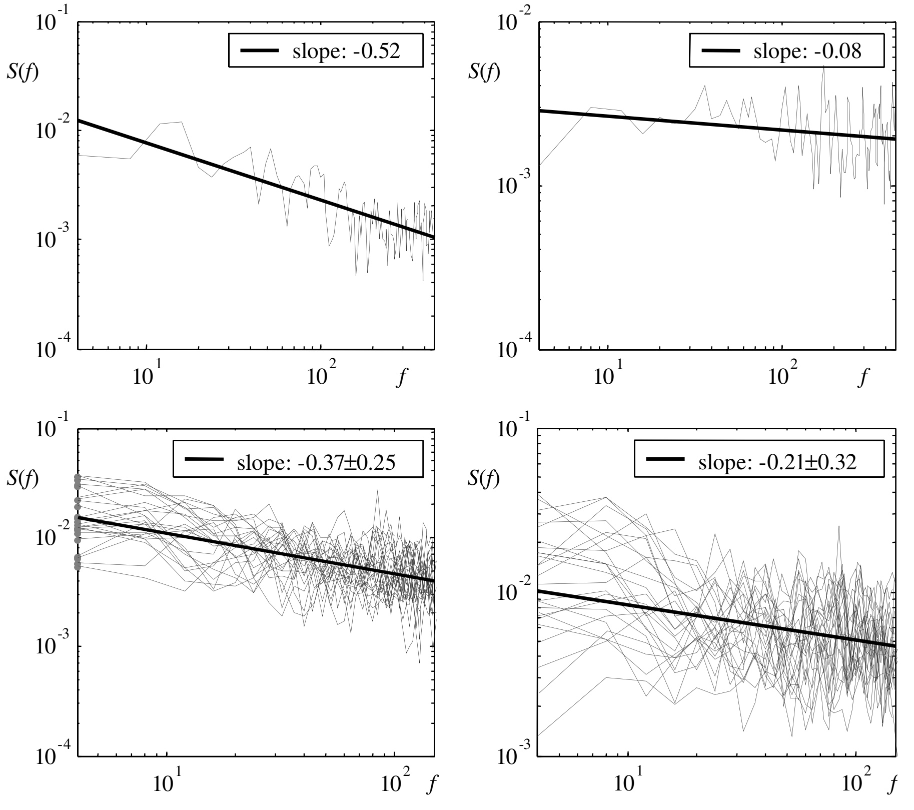

Fig. 9. (Above) Illustrative power spectral density functions for the forces produced by left and right hands (left and right side of figure) from one subject (single trial). The best fit least squares regression line is superimposed on each function. (Below) Linear functions estimated for all subjects and conditions in Experiment 2.

by the least squares regression line superimposed on each function. The total power is greater for the left hand and this is reflected in the higher intercept. The lower half of the figure shows the linear regressions for all twelve subjects.

We expected that KR would reduce the slope and we found this to be the case with mean slopes of −0.088 and −0.497 on trials 1 and 2 respectively. Repeated measures ANOVA on the slopes showed this effect to be statistically reliable ($F(1,11) = 42.749$, $p < 0.001$). There was a significant main effect of hand ($F(1,11) = 7.513$, $p < 0.05$) and a reliable interaction between trial and hand ($F(1,11) = 14.030$, $p < 0.01$). On trial 1 the slope for the RH (for which KR was given) was 0.055 whereas for the LH it was −0.230. On trial 2 RH and LH slopes converged to similar values of −0.492 and −0.501. ANOVA on the intercepts showed significant main effects of trial ($F(1,11) = 41.954$, $p < 0.001$), hand ($F(1,11) = 8.109$, $p < 0.05$) and a significant trial by hand interaction ($F(1,11) = 15.459$, $p < 0.01$). The intercept was more positive for the left (−4.153) than for the right hand (−5.285) on trial 1. However, on trial

2, the intercepts were nearly the same (−3.202 and −3.171 for right and left hands respectively.)

9. Stationarity of the datasets

Estimates of power laws or, more precisely, of $1/f$ characteristics via periodogram methods are known to become less robust when the process under study contains (deterministic) nonstationary components such as linear or parabolic trends, level shifts, etc. (Dang & Molnár, 1999). To support our findings we therefore provide two additional analyses that are either (almost) invariant against such nonstationary effects or, in contrast, are sensitive to drift by showing large deviations of straight-line ($f^{-\beta}$) forms. For the first we compute so-called wavelet-based log-scale diagrams and for the second we utilize the rescaled adjusted range statistics.

To briefly sketch the wavelet analysis for processes with long-range dependencies (see, e.g., Veitch & Abry, 1999) observe that the discrete wavelet transform represents a discrete series $x_1, x_2, x_3, \ldots, x_n$ by a combination of delayed and scaled copies of a mother wavelet function ψ (here the Daubechies wavelet). At scale level s the wavelet coefficients $c_x(s,d)$ are defined as follows:

$$c_x(s,d) = 2^{\frac{s}{2}} \sum_{t=1}^{n} x_t \psi(2^{-s}n - d) \quad \text{with } s = 1,2,\ldots \quad \text{and} \quad d = 1,2,\ldots,2^{-s}n$$

If the process under study is (second order) stationary then one can show analytically that its wavelet coefficients have the following expectation values:

$$E[c_x^2(s,d)] = \int x(f) 2^s \mid \Psi(2^s f)\mid^2 df \propto c_f 2^{\beta s} C_\psi(\beta)$$

where $x(f)$ and $\Psi(f)$ are the power spectra of the time series and the Fourier transform of the mother wavelet, respectively. Numerical studies showed that this form is also valid in the presence of deterministic trends rendering this method unbiased and highly robust for possibly nonstationary processes (Dang & Molnár, 1999). The value $C_\psi(\beta) = \int |f|^{-\beta} |\Psi(f)|^2 df$ is a constant that depends on β and ψ. For a given length n, the available number of wavelet coefficients at octave s is $n_s = 2^s n$ and one can further approximate

$$\log_2 E[c_x^2(s,d)] \approx \log_2 \left[\frac{1}{n_s} \sum_{k=1}^{n_s} \mid c_x(s,d)\mid^2 \right] \propto \beta s + c$$

when abbreviating $c = \log_2(c_f C_\psi(\beta))$. Thus, if x has long-range correlations in terms of $f^{-\beta}$ power law characteristics, then the log-scale diagram, i.e. the graph of $\log_2 E[\cdots]$ versus s, should be linear with slope β – see Veitch and Abry (1999) for more details. Figs. 10 and 11 below show the log-scale diagrams for experiments 1 and 2 and the slope estimates β correspond closely to the estimated slopes of the periodograms (cf. Figs. 7 and 9).

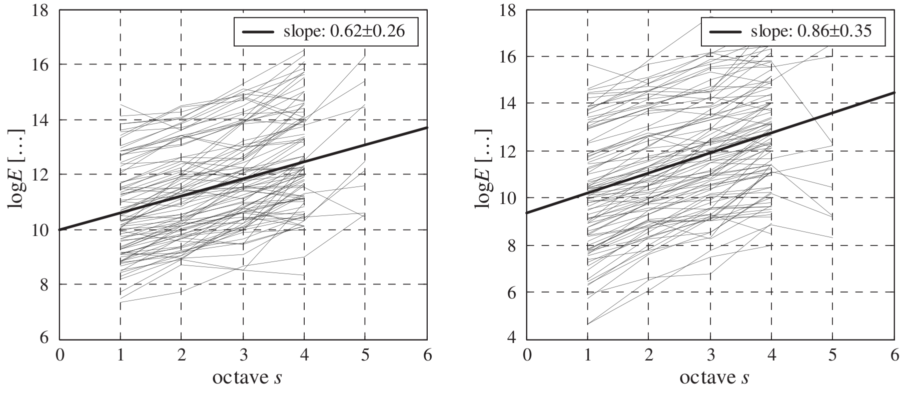

Fig. 10. Log-scale diagrams for load force (left) and grip force (right) data from experiment 1. Compare Fig. 7 lower panels.

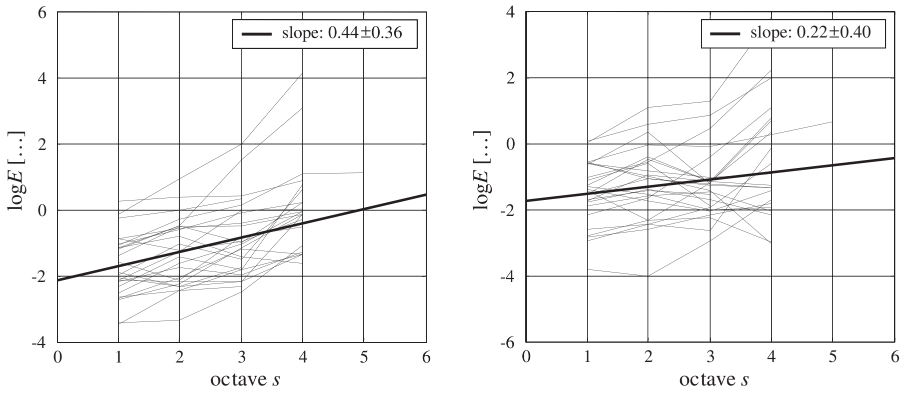

Fig. 11. Log-scale diagrams for left and right hand (left and right panels) force data from experiment 2. Compare Fig. 9 lower panels.

Comparing Figs. 10 and 11 with Figs. 7 and 9 indicates that our periodogram estimates are reliable and, together with the wavelet results, they indicate that the long-range correlations revealed by the $1/f$ functions are not caused by nonstationary processes.

A more traditional alternative (e.g., Dang & Molnár, 1999) to estimate long-range correlations is based on the so-called rescaled adjusted range (R/S) statistics, that, for a time series $x_1, x_2, x_3, \ldots, x_n$ of size n with mean $\bar{x}_n$ and variance $\sigma_n^2 = (\frac{1}{n})\sum_{t=1}^{n}(x_t - \bar{x}_n)^2$ is given by the ratio:

$$\left(\frac{R}{S}\right)_n = \frac{\max\{\omega_t : t = 1, 2, \ldots, n\} - \min\{\omega_t : t = 1, 2, \ldots, n\}}{\sigma_n}$$

Here, we defined $\omega_t = \sum_{k=1}^{t}(x_k - \bar{x}_n)$. Again, analytically one can prove that for a stationary process with long-range dependencies described by a power law with Hurst exponent $H = \frac{1}{2}(\beta + 1)$ for large n, the R/S ratio becomes:

$$E\left[\left(\frac{R}{S}\right)_n\right] \propto \left(\frac{n}{2}\right)^H$$

To compute this ratio for an empirical time series of length N one subdivides the series into K blocks of size N/K which provides $(\frac{R}{S})_n$ ratios at different lags $n = N/K$ and at different starting points t_i (i.e. $t_i = N/K(i-1) + 1$, $i = 1,2 \ldots ,n$) so that K estimates of $\left(\frac{R}{S}\right)_n$ for each value of n are available. Choosing logarithmically spaced values of n ($n < N$) and plotting log $[(\frac{R}{S})_n]$ versus log(n) results in the so-called R/S plot for which a least squares regression line can be fitted to points of the R/S plot (Figs. 12 and 13). The (linear) slope gives an estimate of the Hurst exponent of long-range dependency. Note that

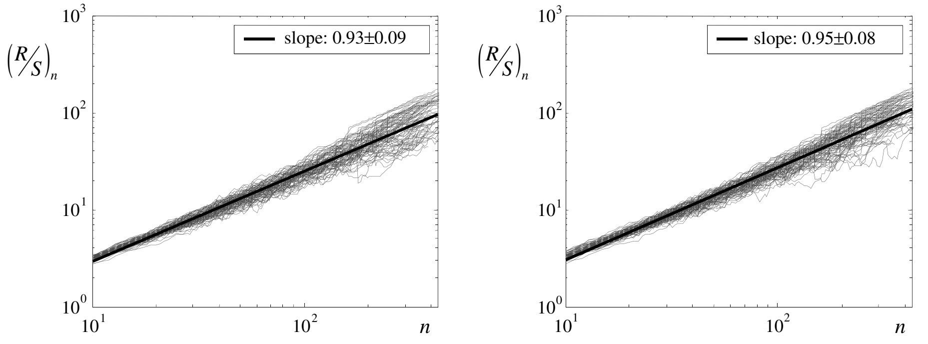

Fig. 12. R/S plot for load force (left) and grip force (right) data from experiment 1. The slope of the regression line estimates the Hurst exponent $H = \frac{1}{2}(\beta + 1)$ of long-range dependency. See text for further explanation and compare Fig. 7 lower panels.

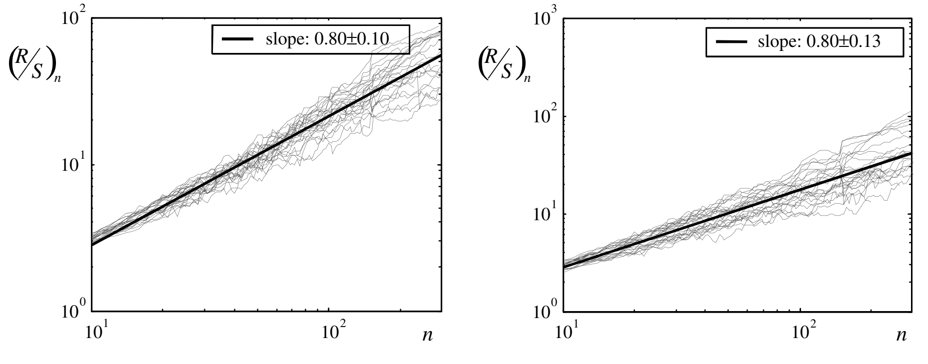

Fig. 13. R/S plot for force data produced by left and right (left and right panels) index fingers in experiment 2. The slope of the regression line estimates the Hurst exponent of long-range dependency (see text). Compare Fig. 9 lower panels.

the smallest values of n should be disregarded because these points are dominated by short-range dependence in the series. Similarly, the high end of the plot should be neglected because only a few points in this region may make the estimate unreliable. We used only values of n in the middle region of the R/S plot to estimate H.

The Hurst exponents in Figs. 12 and 13 correspond to the slope estimates in the periodograms although R/S plots are known to be rather sensitive to nonstationarity by means of strong deviation from straight-line log–log plots (Dang & Molnár, 1999; Viswanthan, Peng, Stanley, & Goldberger, 1997). Again, we can conclude that nonstationary processes do not cause long-range correlations revealed by the experimentally observed $1/f$ functions.

10. Discussion

In the Introduction to this paper we argued that attention in executing repetitive motor tasks operates over a number of time scales and that resulting performance fluctuations can be characterised in terms of spectral characteristics of the associated error variance. In particular we proposed that the degree of sustained attention determines the relative weighting of time scales operating in process control. This led to the hypothesis that the log–log spectral plot might be an indicator of sustained attention in that the greater (i.e. less negative) its slope, the greater the invocation of attention.

In two experiments we found linear log–log relations between power spectral density and frequency, i.e. $1/f$ noise, for maxima in two different force production tasks. In the first experiment subjects made repeated pulls on a fixed manipulandum. The load forces (in the direction of the pull) in these pulses ranged up to 10 N. Each load force pulse required parallel modulation in grip force (normal to the grip surfaces and at right angles to the load force) to keep a stable grip on the manipulandum. Load force and grip force spectral density functions exhibited different slopes; load force had less negative slope than grip force. We attribute this difference in slope to a shorter time scale reflecting attention focussed on the attaining the correct load force with each pull. Why would grip force allow less focussed attention, resulting in longer-time scales in the variability? One possible reason is that, although grip force is important in stabilising grip, there is no cost in allowing it to vary provided the level remains higher than the minimum force to prevent slip.

In the second experiment we examined time scales exhibited in the variation in two series of bilaterally simultaneous force pulses. There were two trials. On the first trial subjects received KR about their right hand force pulses only. On the second trial, no knowledge of results was given. We found zero slope relating power spectral density to frequency in the right hand on the first trial, whereas on the second trial the slope became markedly negative. In contrast the left hand showed an intermediate slope on the first trial and then dropped to the same lower value as the right hand on the second trial. We interpret this result as further evidence that attentional mechanisms associated with response-to-response adjustment of force (in the presence of KR) result in shorter time scales. In contrast, when KR is withdrawn, reduced attention to control processes results in longer-time scales.

As motivation for our approach in the Introduction we noted Gilden (2001) and Van Orden et al. (2003) also documented $1/f$ noise in a number of tasks involving RT. Wagenmakers, Farrell, and Ratcliff (in press) criticised Van Orden et al. for failure to test alternatives to $1/f$ noise, suggesting the functions appear to level off at lower frequencies and, if this were so, the data might be better described by a shorter term ARMA process. However, when Wagenmakers et al. applied ARFIMA (fractionally integrated ARMA) models to Van Orden et al.'s data to determine $1/f$ contribution to ARMA they found the $1/f$ model ranked highest (using an information criterion that weighted residual variance and complexity) in one study and second highest (just behind AR(1)) in the other study. Moreover, in a study of time interval production they showed the ARFIMA model with $1/f$ noise proved a better fit than ARMA(1,1) (Farrell et al., submitted). In considering possible process models Wagenmakers et al. observed that discrete regime switching combined with AR(1) might produce $1/f$ behaviour. [2] Interestingly, Wagenmakers et al. also note that they recently adapted the diffusion model of RT to produce serial dependence by assuming changes in a model parameter on two time scales – short (relating to previous decision) and long (relating to prior knowledge). Taken with our proposal of a contribution of sustained attention on multiple time scales, it is clear that there is a need for explicit testing in future of competing models of the nature of $1/f$ noise in extended series of behavioural observations.

We see a number of links between our work and studies of movement control. A close look at the structure fluctuations in stride variations during gait revealed significant long-term correlations (Hausdorff, Ladin, Peng, Wei, & Goldberger, 1995; Hausdorff et al., 1996; West & Griffin, 1999) that may alter with pathology (Hausdorff, Mitchell, & Firtion, 1997). In standing balance, Collins and DeLuca (1993, 1994) found deviations of the centre of pressure during quiet stance in terms of two different processes that differed in their time scales. [3] A short-term regime was defined by a positively correlated random walk, i.e. a drift away from equilibrium after a perturbation [4] and, this was complemented by a long-term regime composed of a negatively correlated random walk, i.e. a tendency to return to the centre of pressure's equilibrium position.

[2] Kaulakys (1999) also proposed a role for AR(1) in producing $1/f$ noise. However, the AR(1) was used indirectly. Thus, he considered pulse trains whose recurrence times followed an AR(1). Hence, its not only the AR(1) causing $1/f$ but the usage of AR(1) defining recurrence times of a random process that cause $1/f$ characteristics (see Kaulakys, 1999: the process $I(t)$ is defined in Eq. (1) and the AR(1) are given in Eq. (3a,b), the explicit implementation in Eq. (4); the power spectral density of $I(t)$ has an $1/f$ slope).

[3] Collins and DeLuca (1993) studied the mean square displacement of the center of pressure. For any stationary process $\{x_t\}$, however, the mean square displacement is proportional to the auto-correlation function by means of $\langle (x_{t+\tau} - x_t)^2 \rangle = 2\langle x_t^2 \rangle - 2\langle x_{t+\tau} x_t \rangle$. Thus, if the mean square displacement increases with $t^{-\gamma}$, then the auto-correlation and the power spectral density decay accordingly.

[4] Notice that such a positively correlated random walk, or super-diffusive process, requires a dynamical system that follows nonconventional statistics (e.g., Frank, Daffertshofer, & Beek, 2001); see also (Balasubramaniam, Riley, & Turvey, 2000; Riley, Balasubramaniam, & Turvey, 1999; Webber & Zbilut, 1994) for alternative but related approaches to that topic.

Notwithstanding control theoretical interpretations of different time scales, the identification of their source still remain quite speculative (Chen, Ding, & Kelso, 2001). Newell, Liu, and Mayer-Kress (2001) proposed that, in general, action involves at least three cross-coupled levels that operate at distinct time scales. One may think of the microscopic level of physiology, an intermediate level of coordination, and the macroscopic level of performance. Although the first and the last of the levels are clearly distinct, their difference in terms of time scales is not. Time scales in the dynamics of coordination are even less obvious although Newell et al. (2001) suggested the characteristic times of the previously mentioned three levels can be ranked in orders of milliseconds to seconds, minutes to hours and months to years, respectively. Indeed, Huys, Daffertshofer, and Beek (2004a, 2004b) recently studied the acquisition of a complicated coordination task (juggling) and found that the multiform dynamics involved hierarchically ordered time scales reflecting an (abstract) underlying subsystem each. Whether the time scales specify these subsystems (e.g., the frequency and phase adjustment between arm movements, centre of pressure, and point of gaze) or vice versa is, of course, a matter of perspective but their assembly certainly yields long-term correlation patterns similar to the ones reported in the present paper.

In summary we have used modelling and experimental data to show how multiple attentional time scales may underlie 1/*f* noise in the production of extended series of bivariate force pulses.

References

Alessio, E., Carbone, A., Castelli, G., & Frappietro, V. (2002). Second-order moving average and scaling of stochastic time series. *European Physics Journal B, 27*, 197–200.

Bak, P., Tang, C., & Weisenfeld, K. (1987). Self-organized criticality: An explanation of 1/*f* noise. *Physical Review A, 33*, 364–374.

Balasubramaniam, R., Riley, M. A., & Turvey, M. T. (2000). Specificity of postural sway to the demands of a precision task. *Gait Posture, 11*, 12–24.

Box, G. E. P., & Jenkins, G. M. (1976). *Time series analysis: An introduction*. New York: Wiley.

Chatfield, C. (2003). *The analysis of time series. An introduction*. London: Chapman and Hall.

Chen, Y., Ding, M., & Kelso, J. A. S. (2001). Origins of timing errors in human sensorimotor coordination. *Journal of Motor Behavior, 33*, 3–8.

Collier, G. L., & Ogden, R. T. (2004). Adding drift to the decomposition of simple isochronous tapping: An extension of the Wing–Kristofferson model. *Journal of Experimental Psychology: Human Perception and Performance, 30*, 853–872.

Collins, J. J., & DeLuca, C. J. (1993). Open-loop and closed-loop control of posture: A random-walk analysis of center-of-pressure trajectories. *Experimental Brain Research, 95*, 308–318.

Collins, J. J., & DeLuca, C. J. (1994). Random walking during quiet standing. *Physical Review Letters, 73*, 764–767.

Dang, T. D., & Molnár, S. (1999). On the effects of non-stationarity in long-range dependence tests. *Periodica Polytechnica Seria Electrical Engineering, 43*, 227–250.

Ding, M., Chen, Y., & Kelso, J. A. S. (2002). Statistical analysis of timing errors. *Brain and Cognition, 48*, 98–102.

Farrell, S., Wagenmakers, E.-J., & Ratcliff, R. (submitted for publication). ARFIMA time series modeling of serial correlations in human performance.

Frank, T. D., Daffertshofer, A., & Beek, P. J. (2001). Multivariate Ornstein–Uhlenbeck processes with mean field dependent coefficients –Application to postural sway. *Physical Review E, 63*, 1905–1920.

Gilden, D. L. (1997). Fluctuations in time required for elementary decisions. *Psychological Science, 8*, 296–301.

Gilden, D. L. (2001). Cognitive emissions of 1/*f* noise. *Psychological Review, 108*, 33–56.

Gilden, D. L., Thornton, T., & Mallon, M. W. (1995). 1/*f* noise in human cognition? *Science, 267*, 1837–1839.

Handel, P. H., & Chung, A. L. (Eds.). (1993). *Noise in physical systems and 1/f* fluctuations. New York: American Institute of Physics.

Harris, C. M., & Wolpert, D. M. (1998). Signal-dependent noise determines motor planning. *Nature, 394*, 780–784.

Hausdorff, J. M., Ladin, Z., Peng, C. K., Wei, J. Y., & Goldberger, A. L. (1995). Is walking a random walk? Evidence for long-range correlations in stride interval of human gait. *Journal of Applied Physiology, 78*, 349–358.

Hausdorff, J. M., Mitchell, S. L., Firtion, R., et al. (1997). Altered fractal dynamics of gait: Reduced stride-interval correlations with aging and Huntington's disease. *Journal of Applied Physiology, 82*, 262–269.

Hausdorff, J. M., Purdon, P. L., Peng, C. K., Ladin, Z., Wei, J. Y., & Goldberger, A. L. (1996). Fractal dynamics of human gait: Stability of long-range correlations in stride interval fluctuations. *Journal of Applied Physiology, 80*, 1448–1457.

Hayes, M. (1996). *Statistical Digital Signal Processing and Modeling*. John Wiley & Sons.

Huys, R., Daffertshofer, A., & Beek, P. J. (2004a). Multiple time scales and multiform dynamics in learning to juggle. *Motor Control, 8*, 188–202.

Huys, R., Daffertshofer, A., & Beek, P. J. (2004b). Multiple time scales and subsystem embedding in the learning of juggling. *Human Movement Science, 23*, 315–336.

Johansson, R. S., & Westling, G. (1984). Roles of glabrous skin receptors and sensorimotor memory in automatic control of precision grip when lifting rougher and more slippery objects. *Experimental Brain Research, 56*, 550–564.

Kaulakys, B. (1999). Autoregressive model of 1/*f* noise. *Physics Letters A, 257*, 37–42.

Kawai, T., Mihira, Y., Sato, M., & Hayashi, M. (1993). Basis of universal existence of 1/*f* fluctuations. In: P. Handel, A. Chung (Eds.), *Noise in physical systems and 1/f fluctuations*, pp. 639–642.

Madison, G. (2001). Variability in isochronous tapping: Higher-order dependencies as a function of intertap interval. *Journal of Experimental Psychology: Human Perception and Performance, 27*, 411–422.

Madison, G. (2004). Fractal modelling of isochronous serial interval production. *Biological Cybernetics, 90*, 105–112.

Makeig, S., & Inlow, M. (1993). Lapses in alertness: Coherence of fluctuations in performance and EEG spectrum. *Electroencephalography and Clinical Neurophysiology, 86*, 23–35.

Meyer, D. E., Smith, J. E. K., Kornblum, S., Abrams, R. A., & Wright, C. E. (1990). Speed-accuracy tradeoffs in aimed movements: Toward a theory of rapid voluntary action. In M. Jeannerod (Ed.), *Attention and performance. Motor representation and control* (Vol. XIII, pp. 173–226). Hillsdale, NJ: Lawrence Erlbaum.

Meyer, D. E., Smith, J. E. K., & Wright, C. E. (1982). Models for the speed and accuracy of aimed movements. *Psychological Review, 89*, 449–482.

Miller, S. L., Miller, W. M., & McWhorten, P. J. (1993). Extremal dynamics: A unifying physical explanation of fractals, 1/*f* noise, and activated processes. *Journal of Applied Physics, 73*, 2617.

Montroll, E. W., & Shlesinger, M. F. (1982). On 1/*f* noise and other distributions with long tails. *Proceedings of the National Academy of Sciences, 79*, 3380–3383.

Newell, K. M., Liu, Y. T., & Mayer-Kress, G. (2001). Time scales in motor learning and development. *Psychological Review, 108*, 57–82.

Ogden, R. T., & Collier, G. L. (2002). Inference on variance components on autocorrelated sequences in the presence of drift. *Nonparametric Statistics, 14*, 409–420.

Ogden, R. T., & Collier, G. L. (1999). On detecting and modeling deterministic drift in long run sequences of tapping data. *Communications in Statistics, 28*, 977–987.

Pressing, J. (1998a). Error correction processes in temporal pattern production. *Journal of Mathematical Psychology, 42*, 63–101.

Pressing, J. (1998b). Sources for 1/*f* noise in human cognition and performance. Proceedings of the 4th Cognitive Science Conference, Newcastle, Australia.

Pressing, J., & Jolley-Rogers, G. (1997). Spectral properties of human cognition and skill. *Biological Cybernetics, 76*, 339–347.

Rangarajan, G., & Ding, M. (Eds.). (2003). *Processes with long-range correlations. Theory and applications. Lecture notes in physics*. Berlin: Springer.

Riley, M. A., Balasubramaniam, R., & Turvey, M. T. (1999). Recurrence quantification analysis of postural fluctuations. *Gait Posture, 9*, 65–78.

Schuster, H. G. (1996). *Deterministic chaos: An introduction*. New York: John Wiley Sons.

Sergent, V., Hellige, J. B., & Cherry, B. (1993). Effects of responding hand and concurrent verbal processing on time-keeping and motor-implementation processes. *Brain and Cognition, 23*, 243–262.

Shaffer, L. H. (1981). Performance of Chopin, Bach, and Bartok: Studies in motor programming. *Cognitive Psychology, 13*, 326–376.

Sornette, D. (2004). *Critical phenomena in natural sciences. Chaos, fractals, self-organization and disorder: Concepts and tools* (2nd ed.). Berlin: Springer.

Stevens, L. T. (1886). On the time sense. *Mind, 11*, 393–404.

Stuss, D. T., Murphy, K. J., Binns, M. A., & Alexander, M. P. (2003). Staying on the job: The frontal lobes control individual performance variability. *Brain, 126*, 2363–2380.

Stuss, D. T., Shallice, T., Alexander, M. P., & Picton, T. W. (1995). A multidisciplinary approach to anterior attentional functions. *Annals of the New York Academy of Science, 769*, 191–211.

Treisman, M. (1963). Temporal discrimination and the indifference interval: Implications for a model of the "internal clock". *Psychological Monographs, 576*, 1–31.

Van Orden, G. C., Holden, J. G., & Turvey, M. T. (2003). Self-organization of cognitive performance. *Journal of Experimental Psychology: General, 132*, 331–350.

Veitch, D., & Abry, P. (1999). A wavelet-based joint estimator of the parameters of long-range dependence. *IEEE Transactions on Information Theory, 45*, 878–897.

Viswanthan, G. M., Peng, C.-K., Stanley, H. E., & Goldberger, A. L. (1997). Deviations from uniform power law scaling in nonstationary time series. *Physical Review E, 55*, 845–849.

Vorberg, D., & Schulze, H. H. (2002). Linear phase-correction in synchronization: Predictions, parameter estimation, and simulations. *Journal of Mathematical Psychology, 46*, 56–87.

Vorberg, D., & Wing, A. M. (1996). Modelling variability and dependence in timing. In H. Heuer & S. Keele (Eds.). *Handbook of perception and action* (Vol. 2, pp. 181–262). New York: Academic.

Wagenmakers, E.-J., Farrell, S., & Ratcliff, R. (in press). Human cognition and a pile of sand: A discussion on serial correlations and self-organized criticality. *Journal of Experimental Psychology: General*.

Webber, C. L., & Zbilut, J. P. (1994). Dynamical assessment of physiological systems and states using recurrence plot strategies. *Journal of Applied Physiology, 76*, 965–973.

Weissman, M. B. (1988). 1/*f* noise and other slow, nonexponential kinetics in condensed matter. *Review of Modern Physics, 60*, 537–571.

West, B. J., & Griffin, L. (1999). Allometric control, inverse power laws and human gait. *Chaos, Solitons, Fractals, 10*, 1519–1527.

Wing, A. M. (1980). The long and short of timing. In G. E. Stelmach & J. Requin (Eds.), *Tutorials in motor behavior*. Amsterdam: North-Holland Science Publishers.

Wing, A. M. (1992). The uncertain motor system: Perspectives on variability in movement. In D. E. Meyer & S. Kornblum (Eds.), *Attention and performance. Synergies in experimental psychology, artificial intelligence and cognitive neuroscience* (Vol. XIV, pp. 709–744). Cambridge MA: MIT Press.

Wing, A. M., & Kristofferson, A. B. (1973a). The timing of interresponse intervals. *Perception and Psychophysics, 13*, 455–460.

Wing, A. M., & Kristofferson, A. B. (1973b). Response delays and the timing of discrete motor responses. *Perception and Psychophysics, 14*, 5–12.